The Gamma Match

The gamma match was originally invented as a means of feeding vertical monopole antennas for medium wave broadcasts, which were earthed at the base for lightning protection (see Figure 1). Amateurs are more likely to use a gamma match to feed a dipole or loop element, but the basic theory is easier to understand for the case of a $\lambda/4$ monopole. A gamma match often uses a match element that is thinner than the main element, but the maths is much simpler if they are the same thickness, so we will assume that to begin with.

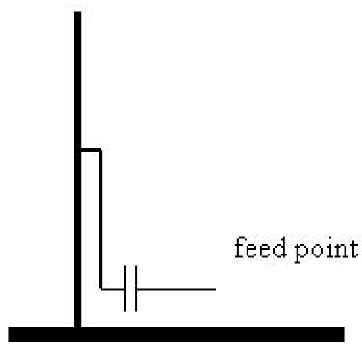

Figure 1: Gamma match feeding grounded vertical monopole

1 Equal Size Elements

The technique used is to consider even and odd mode excitation. For the moment, disconnect the element from ground. It then has two connection points, labelled A and B (Figure 2). Even mode excitation applies the same RF voltage to A and B. Odd mode excitation applies equal but opposite voltages to A and B. To simplify the analysis, the end piece (i.e. the remainder of the antenna) is moved to the centre of the shorting bar. For even mode excitation (Figure 3) the element behaves as a $\lambda/4$ radiator, somewhat thickened for part of its length, so this is sometimes called antenna mode excitation. For the sake of simplicity, assume that the thickening can be ignored. Then the total current flowing into points A and B is given by the ratio of the driving voltage and the radiator feedpoint impedance. As A and B are identical, the current splits equally between them. So we have

$$I_{\mathrm{A-even}} = I_{\mathrm{B-even}} = \frac{V}{2Z_{\mathrm{ant}}} \tag{1}$$

where V is the driving voltage and Z_{ant} is the feedpoint impedance of the antenna element.

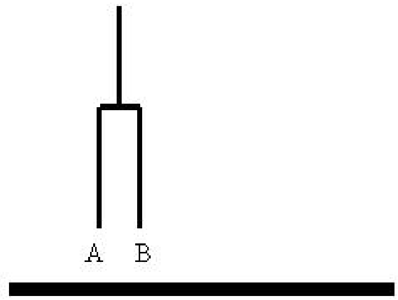

Figure 2: Equal size gamma match

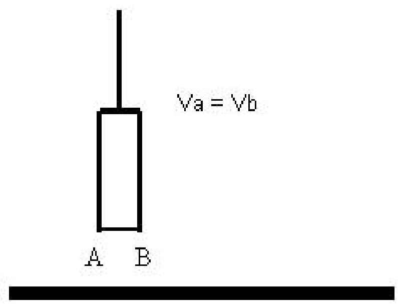

Figure 3: Even mode excitation (antenna mode)

For odd mode excitation (Figure 4, $V_\mathrm{A} = -V$,$V_\mathrm{B} = +V$) the element behaves as a shorted transmission line, with a piece of metal on the end. However we can see, from symmetry, that the voltage at the end is zero so no current flows in the end piece. The shorted transmission line looks like an inductance, as it is usually shorter than $\lambda/4$. The currents flowing into A and B are equal and opposite, and given by the ratio of the driving voltage and the inductive reactance of the shorted line.

$$-I_{\mathrm{A-odd}} = I_{\mathrm{B-odd}} = \frac{2V}{Z_\mathrm{line}} \tag{2}$$

where Z_line is the inductive impedance of the shorted transmission line i.e.

$$Z_\mathrm{line} = jZ_\mathrm{stub} \tan\left(\frac{2\pi l}{\lambda}\right) \tag{3}$$

Z_stub is the characteristic impedance of the twin-wire transmission line formed by the antenna element and the gamma match, l is the length of the gamma match, and of course λ is the wavelength.

$$Z_\mathrm{stub} = 120 \ln\left(\frac{D}{r}\right) = 276 \log_{10}\left(\frac{D}{r}\right) \tag{4}$$

where D is the spacing and r is the element radius. As these measurements only appear as a ratio you can use either metric or imperial units, provided you are consistent.

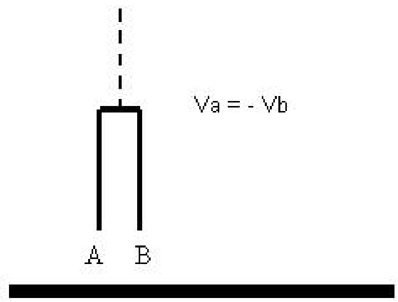

Figure 4: Odd mode excitation (transmission line mode)

The antenna is a linear system so we can use superposition i.e. the result of two simultaneous excitations is just the sum of their individual results. So we can apply the even and odd mode excitations together, and simply add the currents. Point A is then at zero volts i.e. earthed. B is the feed point with a voltage equal to $2V$, and the feed impedance is then found from the ratio of the feed voltage and feed current.

$$Z_{\text{in}} = \frac{2V}{I_{\text{feed}}} \tag{5}$$

but

$$I_{\text{feed}} = I_{\text{B-even}} + I_{\text{B-odd}} = \frac{V}{2Z_{\text{ant}}} + \frac{2V}{Z_{\text{line}}} = 2V\frac{4Z_{\text{ant}} + Z_{\text{line}}}{4Z_{\text{ant}}Z_{\text{line}}} \tag{6}$$

so

$$Z_{\text{in}} = \frac{4Z_{\text{ant}}Z_{\text{line}}}{4Z_{\text{ant}} + Z_{\text{line}}} \tag{7}$$

You may recognise this as the formula for parallel impedances; in this case it is $4Z_{\text{ant}}$ in parallel with Z_{line}. So the effect of an equal size gamma match is to multiply the antenna impedance by four, and then put in parallel with it the inductive impedance of the stub. Note that the length of the stub, l, only affects the stub inductance. Similarly, the spacing only affects the stub characteristic impedance, which in turn affects the stub inductance. If the gamma match extends for the full length of the main element (i.e. $\lambda/4$), then the inductance is infinite and has no effect - this is then simply a folded monopole with four times the impedance of the basic element.

Just to clarify something, the antenna impedance Z_{ant} which appears in the equations is the impedance with the gamma match stub included in the element thickness, but not acting as a gamma match i.e. even mode excitation. This means that the antenna will have a wider bandwidth than it would have without the gamma match, but most of this improvement comes not from the matching but simply because the antenna is effectively thicker. A thin or short gamma match will have little effect. An equal size gamma match, well spaced from the main element and occupying much of its length, will increase bandwidth significantly - this is what happens with the folded dipole.

If the gamma match is shorter than the radiating element then the parallel inductance will make the feed impedance inductive. This can be tuned out either by adding a series capacitor at the feed point, or by shortening the main element so it has some capacitive reactance - see Figure 5. An equal thickness gamma match thus does two impedance transformations: times four for the basic match, then an L-match on the output. The L-match can step the impedance up or down, depending on where the capacitance is placed. If the capacitor is placed in parallel with the feed then this arrangement is sometimes known as an omega match.

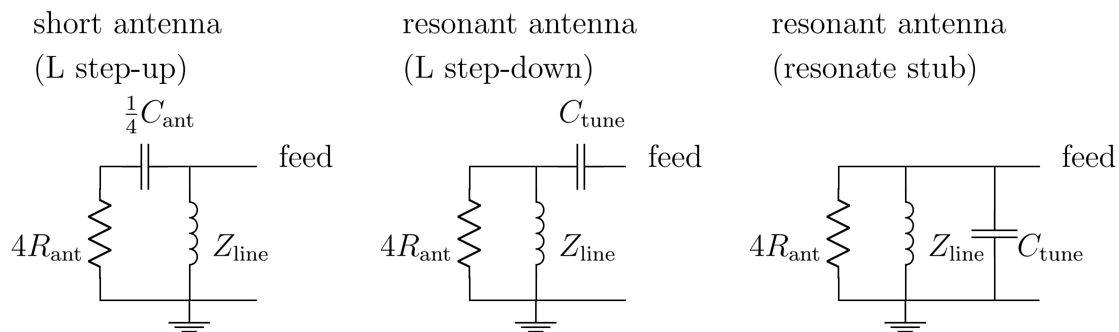

Figure 5: Gamma match effective circuits

2 Higher Ratios

If a bigger impedance step up is needed, then this can be achieved by using a thinner gamma match. This raises the impedance of the transmission line, but also makes it lop-sided so it is neither balanced nor unbalanced. The maths gets messy, although the basic operation remains the same i.e. an impedance step-up combined with an L-match.

The maths is simpler if one considers parallel wires. For example, if there are three wires (two for the antenna, one for the match) then by considering three excitations (all equal, star, +0-) it is found that an impedance step-up of nine times is achieved. A single thicker wire can then replace the two antenna wires.

The maths gets complicated for unequal radii, and full details are given by Balanis[1] in his textbook. I will just give a simplified summary here. The basic impedance step up is n^2, where $n = 2$ for a simple pair with equal sizes as shown above. For unequal

[1] Antenna Theory, C A Balanis, third edition, John Wiley and Sons Inc, 2005: see Chapter 9

sizes:

$$n \simeq 1 + \frac{\ln\left(\frac{D}{r_{\mathrm{G}}}\right)}{\ln\left(\frac{D}{r_{\mathrm{A}}}\right)} \tag{8}$$

where D is the spacing, r_{A} is the radius of the antenna element, and r_{G} is the radius of the stub. As the measurements only appear here as ratios, you can use either metric or imperial units as long as you are consistent. A given impedance ratio can be obtained by many different spacing and radius options. See Figure 6, which has logarithmic axes and shows the spacing and stub radius as a multiple/fraction of the main element radius. For example, an impedance ratio of 10 would come from a

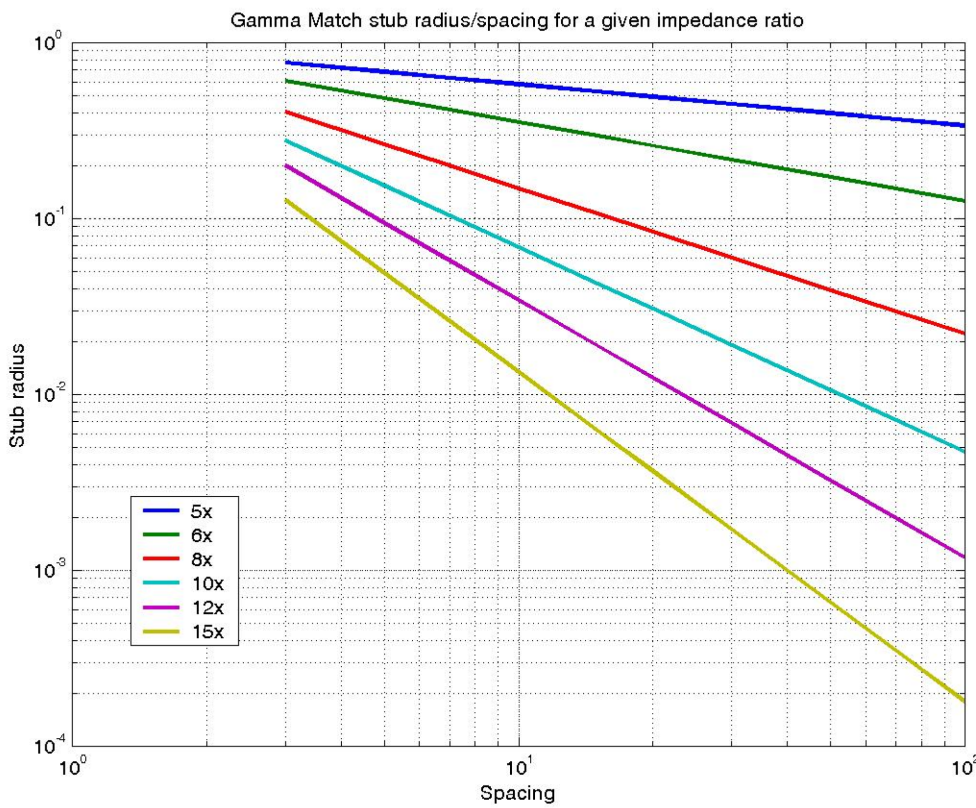

Figure 6: Gamma match impedance ratio

gamma match radius of 0.2 times the main element radius with a spacing of 4 times the main element radius, or stub radius of 0.03 times main element with a spacing of 20 times. Except for the thinnest stubs, changing the spacing has less effect than changing the stub radius.

For unequal radii, the characteristic impedance of the transmission line formed by the stub and main element is

$$Z_{\text{stub}} \simeq 120 \ln \left(\frac{D}{\sqrt{r_{\text{A}} r_{\text{G}}}} \right) \tag{9}$$

A gamma match that is much thinner than the antenna element will give a large impedance multiplication, which may then have to be reduced in the output L-match. This is where the idea of a shorter gamma section giving lower impedance comes from - a short gamma gives a low inductance in the L-match. There are an infinite set of options to give the correct parallel inductance, which may be why adjusting a gamma match seems so tricky.

So, to sum up, the basic impedance transformation is set by the ratio of element and gamma match radius and the spacing, not the length. The length, radius and spacing of the gamma match then have a secondary effect via the inductance in the L-match. The capacitor is optional, and is adjusting for a parallel inductance rather than a series inductance. The capacitor, if present, can be wired in parallel with the feed point rather than in series although a different value will be needed and the final feed impedance will change.

3 Dipole

Up to this point I have been mainly talking about monopoles. What about dipoles and baluns? One option is to use a gamma match for each side (known as a T-match). The calculation is done by treating each side as a monopole i.e. use half the dipole impedance. You get the same result by simply doubling the transmission line impedance instead. The result is a balanced antenna which will need a balun in the usual way.

If a gamma match is used on one side only then things get complicated. This is because the point A we have been treating as zero potential is now the feed point for the other half of the dipole, so can't be zero. I won't bore you with the algebra, but the result is that for a basic impedance step-up of n^2 the voltages come out as

$$V_{\text{A}} = -\frac{1}{2n-1} V_{\text{B}} \tag{10}$$

This means that an impedance ratio of 9 gives an 'earthy end' voltage of 20% of the 'live end' voltage. Does this confirm that a balun is not necessary? It all depends on how fussy you are. Once impedance ratios get up above, say, 30 then a balun can be dispensed with but this sort of impedance ratio will only be seen when feeding a small loop. A gamma match on the driven element of a Yagi-Uda array will generally need lower ratios so a balun may still needed. One way to get round this is to design for a much higher basic ratio (i.e. thin stub, close spacing), then drop the feed impedance with a short (i.e. low inductance) stub tuned out with a large series capacitor. This may be how some gamma matches are (accidentally?) designed.

Using a gamma match on only half of a dipole means that only half the dipole impedance is transformed. The other half then acts as a counterpoise. As the dipole impedance was already low (otherwise you would not be using a gamma match) this counterpoise will be quite effective, but perhaps not so good as to completely eliminate the need for a balun.